Supersonic Flutter of Heated Circular Cylindrical Shells with Temperature-Dependent Material Properties

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This paper presents an analysis of the supersonic flutter of thin circular cylindrical shells. Galerkin's method is used to reduce the nonlinear shell equations to ordinary differential equations which are then solved asymptotically according to the method of Krylov-Bogoliubov. Nonstationary, axially varying shell wall temperatures are considered. It is shown that by allowing the material properties to vary with temperature, the flutter boundaries may be reduced by as much as 25 to 30%. Essential qualitative and quantitative changes in vibration and stability are introduced by temperature-dependent effects.

Introduction

IT has been shown[1] that the critical gas velocity at linear flutter of an infinite circular cylindrical shell may decrease by as much as 50% when the shell is exposed to a uniformly distributed, linearly transient temperature field. It has also been found[2] that the frequency of linear free vibrations of plates may be reduced up to 10% if the plate is exposed to a nonuniformly distributed, stationary temperature field.

Here, we want to study the supersonic flutter of a finite circular cylindrical shell exposed to axially nonuniform and exponentially transient temperatures due to, e.g., convective heating. Experimental observations of such shells have shown[3] that the flutter is strongly influenced by nonlinear effects. Therefore, we begin by deriving nonlinear differential equations for a circular cylindrical shell with temperature-dependent material properties. This is a special case of the equations given in Ref. 1 for general shells of revolution.

Following the methods developed for isothermal problems of this type,[4-6] we expand the radial deflections of the shell in functions satisfying the prescribed boundary conditions. Using Galerkin's variational method, the problem is then reduced to two coupled nonlinear ordinary differential equations in the time variable. These equations are solved asymptotically according to the method of Krylov-Bogoliubov. The solutions give the flutter boundaries as well as the transient variations of amplitudes and frequencies under the influence of the prescribed temperatures.

Basic Equations

Let the median surface of the cylindrical shell be described by the longitudinal coordinate ξ and the circumferential coordinate η. Let the coordinate ζ be directed along the outward normal of the median surface. Further, let (u,v,w) be the corresponding displacement vector in the (ξ,η,ζ) system.

The shell is exposed to a uniform supersonic flow parallel to the axial direction. No other external loads are applied to the shell. Inertia forces in the plane of the shell will be neglected. If the Mach number of the flow is sufficiently high, then the resulting aerodynamic forces on the surface of the shell may be approximated by the linear piston theory including a curvature correction term.[7] The ends of the shell are assumed to be elastically supported but will be free to move both radially and axially due to the influence of wall temperature.

To facilitate the calculations, the quasistatic deflections have been neglected. This is motivated by the fact that the ends of the shell are free to expand with the temperature. Therefore, at realistic axial temperature variations, the quasistatic deflections should be small compared to the dynamic ones.

Then, using generally known simplifications from thin shell theory, the nonlinear differential equations for a circular cylindrical shell with temperature-dependent material properties are

$$\frac{\delta^3}{12}\frac{\partial^2}{\partial\xi^2}\left(\frac{\nu E}{1-\nu^2}\frac{\partial^2 w}{\partial\eta^2}+\frac{E}{1-\nu^2}\frac{\partial^2 w}{\partial\xi^2}\right)+\frac{\delta^3}{6}\frac{\partial^2}{\partial\xi\partial\eta}\times\left(\frac{E}{1+\nu}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)+\frac{\delta^3}{12}\frac{\partial^2}{\partial\eta^2}\left(\frac{E}{1-\nu^2}\frac{\partial^2 w}{\partial\eta^2}+\frac{\nu E}{1-\nu^2}\frac{\partial^2 w}{\partial\xi^2}\right)$$
$$+\frac{1}{R}\frac{\partial^2\Phi}{\partial\xi^2}-\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2\Phi}{\partial\eta^2}+2\frac{\partial^2 w}{\partial\xi\partial\eta}\frac{\partial^2\Phi}{\partial\xi\partial\eta}-\frac{\partial^2 w}{\partial\eta^2}\frac{\partial^2\Phi}{\partial\xi^2}$$
$$+\rho a U\frac{\partial w}{\partial\xi}+\rho a\frac{\partial w}{\partial t}-\frac{\rho a^2}{2R}w+\delta\rho_s\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

$$\frac{1}{\delta}\frac{\partial^2}{\partial\xi^2}\left(\frac{1}{E}\frac{\partial^2\Phi}{\partial\xi^2}-\frac{\nu}{E}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{1}{\delta}\frac{\partial^2}{\partial\eta^2}\left(\frac{1}{E}\frac{\partial^2\Phi}{\partial\eta^2}-\frac{\nu}{E}\frac{\partial^2\Phi}{\partial\xi^2}\right)+\frac{2}{\delta}\frac{\partial^2}{\partial\xi\partial\eta}\left(\frac{1+\nu}{E}\frac{\partial^2\Phi}{\partial\xi\partial\eta}\right)-\frac{1}{R}\frac{\partial^2 w}{\partial\xi^2}$$
$$-\left(\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2+\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}=0 \tag{2}$$

In these equations δ is the thickness of the shell wall, E is Young's modulus, ν is Poisson's ratio, and Φ is the usual stress function. R is the radius of the shell. Also ρ is the density of the gas stream, U its velocity, a the speed of sound, and ρ_s the density of the shell material.

Solution of the Equations

A solution w is assumed in the form

$$w=\delta\left(f\sin\frac{\pi\xi}{L}+g\sin\frac{2\pi\xi}{L}\right)\cos\frac{m\eta}{R}-\delta^2\frac{m^2}{4R}\left(f\sin\frac{\pi\xi}{L}+g\sin\frac{2\pi\xi}{L}\right)^2 \tag{3}$$

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where the last term must be included in order to satisfy the periodic continuity condition on the circumferential displacement v.[4] In the temperature-independent case, substitution of Eq. (3) into the compatibility equation [Eq. (2)] allows the latter to be solved for the particular solution Φ, while the complementary solution is assumed to vanish.[5,6] Here, the variable coefficients of Eq. (2) make the problem more complicated. It is assumed that Φ has the same form as in the temperature-independent case, i.e.,

$$\Phi = \cos\frac{m\eta}{R}\sum_{n=1}^{6} F_n \sin\frac{n\pi\xi}{L} + \cos\frac{2m\eta}{R}\sum_{n=0}^{4} F_{n+7}\cos\frac{n\pi\xi}{L} + \cos\frac{m\eta}{R}\sum_{n=0}^{4} F_{n+12}\cos\frac{n\pi\xi}{L} \tag{4}$$

Then the parameters F_n are solved in terms of the nonlinear deflection parameters f and g through substituting Eqs. (3) and (4) into Eq. (2) and using Galerkin's method with $\partial\Phi/\partial F_n$ as weighting functions. When the stress function Φ has been found, it is introduced into Eq. (1) together with the assumed form of w and Galerkin's method is again used to derive nonlinear ordinary differential equations for the functions f and g. Now $\partial w/\partial f$ and $\partial w/\partial g$ are used as weighting functions. The equations are given in the Appendix. The coefficients depend on a parameter $\mu = m^4\delta^2/R^2$ so that $\mu = 0$ linearizes the equations. Note that for transient temperatures, the coefficients are time dependent so that there are no simple solutions even if the nonlinear terms are neglected.

Equations (A1) and (A2) are valid for arbitrarily variable material properties. To simplify the problem we want to restrict the analysis to tangentially uniform properties. Therefore, we introduce a temperature field where through convective heating, for example, the shell wall temperature increases exponentially with a time constant τ according to

$$\theta = [\theta_0 + \theta_l \cos(\pi\xi/L)][1 - e^{-(t/\tau)}] \tag{5}$$

The axial temperature variation in Eq. (5) was chosen so as to simplify the calculations. To this end we shall also assume that the material properties E and ν are only moderately temperature dependent. Thus, they are introduced in the form $E = E_0 + \epsilon E_l \theta$, where ϵ is a small arbitrary parameter. Linearizing with respect to ϵ facilitates the calculation of the coefficients.

Experiments indicate[5] that the flutter motion is nearly sinusoidal in time so that the solutions may be sought in the form

$$f = F\cos\psi \tag{6}$$

$$g = G\cos(\psi + \phi) \tag{7}$$

where F, G, and ϕ are slowly varying functions of time which may be found with the method of Krylov-Bogoliubov retaining first-order terms in ϵ and μ.

The result is four coupled nonlinear and first-order differential equations in F, G, ψ, and ϕ. They provide solutions for the transient variation of the amplitudes, the phase angle, and the frequency. For steady-state oscillations they also provide the flutter boundaries.

Results and Discussion

The shell properties, flow conditions, and loadings have been chosen quite arbitrarily as

$\delta = 1$ mm	$R = 0.5$ m
$\rho_s = 8150$ kg/m^3	$L = 3$ m
$E_0 = 2.12\cdot 10^{11}$ N/m^2	$\nu_0 = 0.30$
$\epsilon E_l = -7.4\cdot 10^7$ N/m^2°C	$\epsilon\nu_l = 5.8\cdot 10^{-5}$ 1/°C
	$\tau = 9.2$ s

The temperature of the freestream was taken as 1500 K with the wall temperature increasing by $\theta_0 = 900$°C.

In linear theory, the flutter boundary is defined by a critical velocity U_c. Thus, disturbances of the modal amplitudes are damped out with time for velocities below U_c and increase with time for velocities greater than U_c. The calculations have been carried out for a circumferential wave number $m = 11$ corresponding to minimum flutter velocity U_{c0} for the unheated shell in linear theory. This wave number is assumed to remain constant during the heating up of the shell.

With increasing shell wall temperature, the flutter velocity decreases in correspondence with the weakening of the material properties. This temperature dependence of the flutter velocity is shown in Fig. 1. For a temperature field, according to Eq. (5), the flutter boundary moves from point A to point B during uniform heating with $\theta_0 = 900$°C and to point C if there is also an axial temperature variation $\theta_l = 50$°C. Thus, the axial temperature variation has a stabilizing effect. This stabilizing effect is independent of which end of the shell is heated as long as the stiffness variation is antisymmetric. It then follows that the part of the elastic energy that comes from the stiffness variation is evenly distributed between the two basic flutter modes of Eq. (3), which is easily seen by considering the corresponding beam problem as derived from Eqs. (1) and (2). That stiffness variations lead to higher stability is also observed in other cases as, e.g., buckling.

When nonlinear terms are included in the analysis, stability depends not only on the aerodynamic conditions but also on the size of the disturbances themselves. Furthermore, unstable oscillations may eventually develop into steady-state vibrations with finite amplitudes, so-called limit cycles.

Nonlinear flutter boundaries are shown in Fig. 2 for a freestream static pressure of 20 kPa. In linear theory the curves would be straight vertical lines. Numerical stability studies[5] have shown that only those parts of the curves in Fig. 2 which have positive slope represent stable limit cycles. Thus,

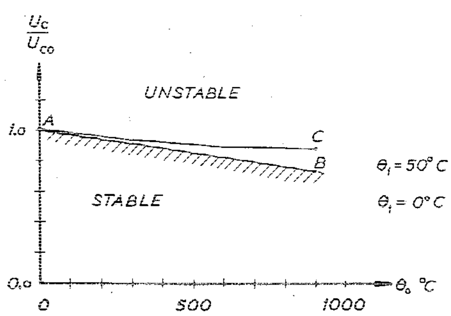

Fig. 1 Normalized flutter velocity vs average temperature θ_0 and axial temperature variation θ_l.

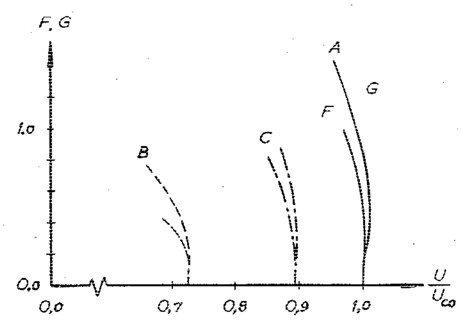

Fig. 2 Modal limit cycle amplitudes vs normalized velocity at $p = 20$ kPa.

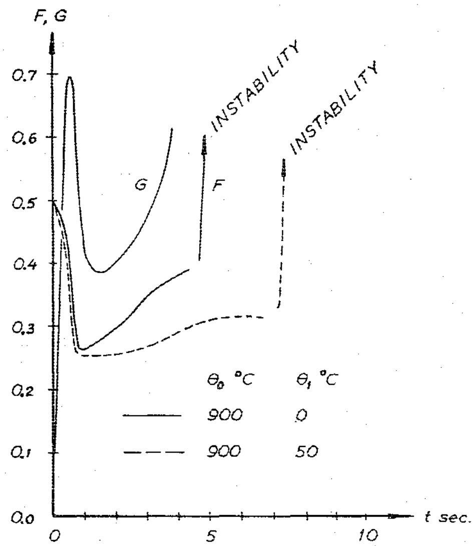

Fig. 3 Modal amplitudes vs time at normalized velocity 0.975 and initial phase angle $\phi_0 = 0$.

for all initial conditions to the left of and below the curves the oscillations will be damped out with time. For all other initial conditions they will grow, either up to the curves or to infinitely large values.

During heating of the shell, the flutter boundaries move to lower velocities. For small amplitudes, this movement may be predicted by linear theory. Thus, curves A, B, and C in Fig. 2 correspond to the points indicated in Fig. 1.

Obviously, initially stable oscillations may change over to unstable ones at transient heating of the shell wall. Such courses of events are shown in Fig. 3. Although the amplitudes may disappear almost completely, they still remain latent and may suddenly rise to very large values or to a bounding limit cycle. Axial temperature variations tend to delay the instability.

For different initial values, the two modal amplitudes F and G move together and then follow each other asymptotically. At equal initial values, they follow each other very closely. Higher initial values lead to a more rapid instability, as can be concluded from Fig. 2.

The phase angle rapidly takes on values close to those of a limit cycle at the given gas velocity. It then remains relatively constant until instability occurs when it falls rapidly towards a limiting value of 90 deg. The angular frequency also rapidly assumes values characteristic of a limit cycle, remains relatively constant for a while, and then rapidly diminishes as the amplitudes take on large values.

The structural damping has been neglected here. It would move the points of vertical tangency of the curves in Fig. 2 to somewhat higher velocities. This would lead to an increased damping of the curves in Fig. 3, but it would not change the results qualitatively.

Conclusions

The effect of transient heating is to alter the flutter boundaries of thin-walled cylindrical shells significantly. The present paper shows that by allowing the material properties to vary with temperature (and in the axial direction) the flutter boundary may be reduced by as much as 25 to 30%. The effect of such thermal variations has been neglected in most previous studies, and it may partially explain the lack of agreement between theory and experiment for flutter of circular cylindrical shells.

Appendix

The nonlinear differential equations for the functions $f(t)$ and $g(t)$ will be

$$\left(1+\frac{3}{8}\mu f^2+\frac{1}{4}\mu g^2\right)\frac{d^2f}{dt^2}+\frac{1}{2}\mu fg\frac{d^2g}{dt^2}+\left(\frac{a\rho}{\delta\rho_s}+\mu\frac{3a\rho}{8\delta\rho_s}f^2\right.$$
$$\left.+\mu\frac{a\rho}{4\delta\rho_s}g^2+\frac{3}{8}\mu f\frac{df}{dt}\right)\frac{df}{dt}+\mu\left(\frac{a\rho}{2\delta\rho_s}fg+\frac{1}{4}f\frac{dg}{dt}\right)\frac{dg}{dt}$$
$$+a_1f+a_2g+a_3f^2+a_4fg+a_5g^2+a_6f^3+a_7f^2g+a_8fg^2$$
$$+a_9g^3+a_{10}f^4+a_{11}f^3g+a_{12}f^2g^2+a_{13}fg^3+a_{14}g^4+a_{15}f^5$$
$$+a_{16}f^4g+a_{17}f^3g^2+a_{18}f^2g^3+a_{19}fg^4+a_{20}g^5=a_{21} \quad \text{(A1)}$$

$$\left(1+\frac{3}{8}\mu g^2+\frac{1}{4}\mu f^2\right)\frac{d^2g}{dt^2}+\frac{1}{2}\mu fg\frac{d^2f}{dt^2}+\left(\frac{a\rho}{\delta\rho_s}+\mu\frac{3a\rho}{8\delta\rho_s}g^2\right.$$
$$\left.+\mu\frac{a\rho}{4\delta\rho_s}f^2+\frac{3}{8}\mu g\frac{dg}{dt}\right)\frac{dg}{dt}+\mu\left(\frac{a\rho}{2\delta\rho_s}fg+\frac{1}{4}g\frac{df}{dt}\right)\frac{df}{dt}$$
$$+b_1f+b_2g+b_3f^2+b_4fg+b_5g^2+b_6f^3+b_7f^2g+b_8fg^2$$
$$+b_9g^3+b_{10}f^4+b_{11}f^3g+b_{12}f^2g^2+b_{13}fg^3+b_{14}g^4+b_{15}f^5$$
$$+b_{16}f^4g+b_{17}f^3g^2+b_{18}f^2g^3+b_{19}fg^4+b_{20}g^5=b_{21} \quad \text{(A2)}$$

The coefficients of these equations are rather complicated and will not be given here. They may be found in Ref. 8.

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